



11.4 An interpretation of the inner product

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Introduction

- In this topic, we will
 - Interpret the inner product with the adjoint





Matrix-vector multiplication

- Recall that the inner product for real n -dimensional vectors is

$$\langle \mathbf{u}, \mathbf{v} \rangle = \sum_{k=1}^n u_k v_k$$

- For complex vectors, the definition is

$$\langle \mathbf{u}, \mathbf{v} \rangle = \sum_{k=1}^n u_k^* v_k$$





The inner product of $A\mathbf{u}$ and $\mathbf{v}$

- Recall that when we perform a matrix-vector product, we are multiplying a row of the matrix times the vector element-wise and adding

$$A\mathbf{u} = \begin{pmatrix} 3 & 1 & -1 & 2 \\ \underline{2} & \underline{4} & \underline{2} & \underline{1} \\ -2 & 1 & 5 & -3 \end{pmatrix} \begin{pmatrix} \underline{1} \\ \underline{-1} \\ 2 \\ 3 \end{pmatrix}$$

- If we interpret the *transpose* of a vector to be a row vector, then it follows that

$$\langle \mathbf{u}, \mathbf{v} \rangle = \mathbf{u}^T \mathbf{v} = (\underline{2} \quad \underline{4} \quad \underline{2} \quad \underline{1}) \begin{pmatrix} \underline{1} \\ \underline{-1} \\ 2 \\ 3 \end{pmatrix}$$





The inner product of $A\mathbf{u}$ and $\mathbf{v}$

- If the matrices are complex, there is no change to how matrix-vector multiplication is performed

$$A\mathbf{u} = \begin{pmatrix} 3-2j & 1+3j & -1-5j & 2+2j \\ \cancel{2-4j} & \cancel{4+5j} & \cancel{2-6j} & \cancel{1+3j} \\ -2-2j & 1+7j & 5+4j & -3+5j \end{pmatrix} \begin{pmatrix} \cancel{1+4j} \\ -1-2j \\ 2-3j \\ 3-5j \end{pmatrix}$$

- If, however, we transpose the vector and conjugate the entries, we get the complex inner product

$$\langle \mathbf{u}, \mathbf{v} \rangle = \mathbf{u}^* \mathbf{v} = (\cancel{2-4j} \ \cancel{4+5j} \ \cancel{2-6j} \ \cancel{1+3j}) \begin{pmatrix} 1+4j \\ -1-2j \\ 2-3j \\ 3-5j \end{pmatrix}$$





Summary

- Following this topic, you now
 - Understand that an alternative interpretation of the inner product is to define the *adjoint* of a vector and mimic the product of a matrix and a vector

$$\langle \mathbf{u}, \mathbf{v} \rangle = \mathbf{u}^* \mathbf{v}$$





References

- [1] https://en.wikipedia.org/wiki/Hermitian_adjoint



Acknowledgments

None so far.





Colophon

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